

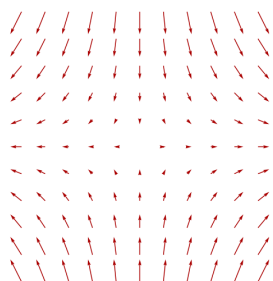
Proposition de sujet de TER de mathématiques :  
**The study of fingerprints, an application of line fields**

**Supervision:** N. Boizot\*

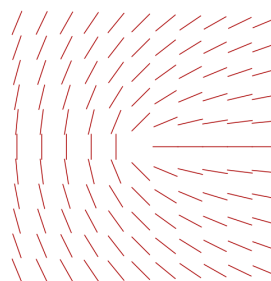
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**Thematic: geometry, numerical analysis.**

This work deals with the study of certain features of line fields defined on the plane. In particular, line fields singularities will be considered. A line field is an application that maps a point of the plane to a line. This latter line being, for instance, represented by an angular value in  $[0, \pi[$ .



(a) A vector field

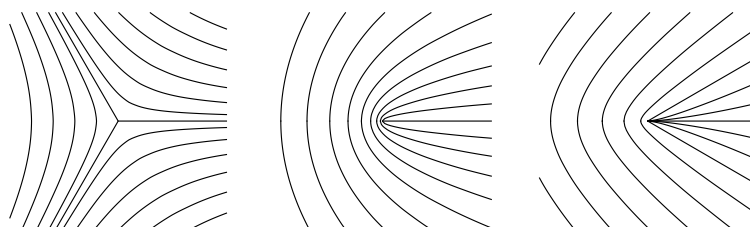


(b) A line field

Although overlooked in comparison with vector fields, this mathematical objet is relevant in order to model a wide variety of physical phenomena (fingerprints, liquid crystals in their nematic phase, visual cortex V1, delicate french pastries,...).



Independently from the considered phenomenon, three types of singularities are identified. These are represented below.



The study of these singularities led the authors of [2] to propose a representation of line fields by means of a pair of vector fields. At each point of the plane, the direction of the line field is given by the bisector of the angle formed by the two corresponding vectors. In this representation, the singularities of the line field coincide with the singularities of either vector fields.

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\*LIS, X165a, boizot@univ-tln.fr

### **Proposed tasks.**

First, the candidate is expected to familiarize with the notion of line fields and study the bisector representation.

Second, article [1] will be studied with the aim of reproducing, and possibly extending, the applications to the study of fingerprints that are presented.

An effective implementation of some of the encountered algorithms is expected.

### **References**

- [1] Y. Wang and J. Hu, *Estimating Ridge Topologies with High Curvature for Fingerprint Authentication Systems*, 2007 IEEE International Conference on Communications, Glasgow, 2007, pp. 1179-1184. doi: 10.1109/ICC.2007.200
- [2] U. Boscain, L. Sachelli and M. Sigalotti, *Generic singularities of line fields on 2D manifolds*, *Differential Geometry and its Applications*, 49, 2016, pp. 326-350.

**Both articles are available upon request.**