Program: DYNQUA 9 - Toulon University - February, 22-24

Wednesday, February 22

13:30-14:00: Opening

14:00-14:45: Jérémy FAUPIN
Scattering theory for Lindblad operators 1

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The scattering matrix and its spectrum in the semiclassical limit 2

15:15-15:45: Guillaume IDELON-RITON
On some scattering properties of the massive Dirac fields in the Schwarzschild-Anti-de Sitter spacetime 3

Coffee Break

16:15-17:00: Stéphane NONNENMACHER
Spectral correlations for randomly perturbed nonselfadjoint operators 4

17:05-17:50: Laurent BRUNEAU
Conductances, ac spectrum and periodic approximants of 1d-systems 5

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Stable Determination outside a cloaking region of two time-dependent coefficients in an hyperbolic equation from Dirichlet to Neumann map 10

14:35-15:20 Konstantin PANKRASHKIN.
A condition for the absence of resonances at the bottom of essential spectrum for waveguide junctions 11

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Asymptotic expansion of eigenvalues for the MIT bag model 13

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Titles and abstracts

Talk 1. [J. Faupin]
Title: Scattering theory for Lindblad operators
Abstract: In this talk, I will consider a quantum particle interacting with a target. The target is supposed to be localized and the dynamics of the particle is supposed to be generated by a Lindbladian acting on the space of trace class operators. I will discuss scattering theory for such models associated to a Lindblad operator. First, I will consider situations where the incident particle is necessarily scattered off the target, next situations where the particle may be captured by the target. An important ingredient of the analysis consists in studying scattering theory for dissipative operators on Hilbert spaces.

This is joint work with Marco Falconi, Juerg Froehlich and Baptiste Schubnel

Talk 2. [M. Ingremeau]
Title: The scattering matrix and its spectrum in the semiclassical limit.
Abstract: Consider a Schrödinger operator $P_h = -h^2\Delta + V$, where $V \in C_c^\infty(\mathbb{R}^d)$. A solution of $P_h f = f$ may always be written as the sum of an incoming and an outgoing part. The scattering matrix is the operator which maps the incoming part to the outgoing part. We will describe some of the properties of S_h, and of its spectrum, in the semiclassical limit $h \to 0$.

Talk 3. [G. Idelon-Riton]
Title: On some scattering properties of the massive Dirac fields in the Schwarzschild-Anti-de Sitter spacetime.
Abstract: I will first introduce the Schwarzschild-Anti-de Sitter spacetime and give some of its geometrical properties. Then, I will talk about the Dirac equation in this geometry and quickly solve the Cauchy problem which is not straightforward in this type of geometry. To analyze the solution obtained, the point of view that is taken then is that of scattering theory. First, I will give a result about asymptotic completeness which correspond to compare our dynamic to a simpler one in the asymptotic regions of our spacetime for large time. This result is proven using Mourre theory and velocity estimates. I will then talk about the local energy decay for these fields which means properties of these fields in bounded region for large time. By means of the construction of quasimodes, it is possible to obtain a lower bound on this decay. I will then introduce some tools, linked to resonances, in order to obtain an upper bound.

Talk 4. [S. Nonnenmacher]
Title: Spectral correlations for randomly perturbed nonselfadjoint operators
Abstract: We are interested in the spectrum of semiclassical nonselfadjoint operators. Due to a strong pseudospectral effect, a tiny perturbation can dramatically modify the spectrum of such an operator. Hager & Sjöstrand have thus considered adding small random pertubations, and proved that the eigenvalues of the perturbed operator typically spread over the classical spectrum, satisfying a probabilistic Weyl's law in the semiclassical limit.

Beyond this Weyl's law, we investigate the correlations between the eigenvalues, at microscopic distances. In the case of 1-dimensional operators, these correlations depend on the structure of the energy shell of the unperturbed operator (a finite set of points), and of the type of perburbation (random matrix vs. random potential), but otherwise enjoy a form of universality, where the central object is the Gaussian Analytic Function (GAF), a family of random entire functions. The GAF was originally introduced in the context of Quantum Chaos in the 1990s, in order to describe the statistical properties of 1D chaotic eigenfunctions. In the present model the GAF (and its variants) rather arise through the spectral determinant of our randomly perturbed operator.

(This is a joint work with Martin Vogel (Orsay).)

Talk 5. [L. Bruneau]
Title: Conductances, ac spectrum and periodic approximants of 1d-systems.
Abstract: The dynamical characterization of the spectral types (pure point, singular continuous, absolutely continuous) of a quantum Hamiltonian h is a rather subtle question. In this talk we focus on the well established heuristics that the ac spectrum corresponds to the set of energies at which the described system exhibits transport. More precisely, we address the question of the characterization of the ac spectrum of one-dimensional Schrödinger operators $h = -\Delta + v$ acting on $l^2(Z_+)$ in terms of the limiting behavior of the conductance of the associated finite samples. The finite samples are defined by restricting h to a finite interval $[1, L] \cap Z_+$ and the conductance refers to the charge current across the sample in the open quantum system obtained by attaching independent electronic reservoirs to the sample ends. We will consider two notions of conductance, one of which, the Thouless conductance, is closely related to the notion of periodic approximant.
(Joint work with V. Jaksic, Y. Last and C.-A. Pillet).

Talk 6. [S. Bachmann]
Title: The adiabatic theorem for quantum spin systems.
Abstract: In this talk, I will present the proof of an adiabatic theorem for ground state projections of a smooth family of many-body gapped quantum systems. Crucially, the diabatic error is uniformly bounded in the volume of the interacting system. As an application, Kubo's formula of linear response theory can be obtained in the thermodynamic limit.

Talk 7. [D. Gobin]
Title: Inverse scattering problem at fixed energy on asymptotically hyperbolic Stäckel manifolds.
Abstract: In this talk we will study an inverse scattering problem on three-dimensional Stäckel manifolds with the topology of a toric cylinder and endowed with an asymptotically hyperbolic structure in the two radial ends. We will see that the Stäckel structure, introduced in 1891, associated with an additional condition, called the Robertson condition, implies the separability of the Helmholtz equation and allows us to reduce our problem to an one-dimensional problem. Moreover the asymptotically hyperbolic structure allows us to define the scattering matrix for every non-zero energies. We will then show that the knowledge of the scattering matrix at one fixed energy is enough to determine uniquely the metric up to classical invariances.

Talk 8. [J.-B. Bru]
Title: Lieb–Robinson Bounds for Multi–Commutators and Applications to Response Theory.
Abstract: I will explain how the usual Lieb–Robinson bounds for commutators can be generalized to multi–commutators. Then, I will show how bounds for multi–commutators of an order higher

than two can be used to study linear and non–linear responses of interacting fermions to external perturbations.

Talk 9. [D. Carpentier]
Title: Topological properties of periodically driven states.
Abstract: In the last decades, a great deal of effort has been devoted to the description of topological insulators : equilibrium gapped phases whose ground state is characterized by a topological invariant, and which possess metallic states at their surface. In this talk, I will describe analogous out-of-equilibrium periodically driven quantum states, described by a gapped evolution operator. I will review the possible topological properties of such a state, and some of the expected physical consequences.

Talk 10. [I. Ben Aïcha]
Title: Stable Determination outside a cloaking region of two time-dependent coefficients in an hyperbolic equation from Dirichlet to Neumann map.
Abstract: In this work, we treat the inverse problem of determining two time-dependent coefficients appearing in a dissipative wave equation, from measured Neumann boundary observations. We establish in dimension $n \geq 2$, stability estimates with respect to the Dirichlet-to-Neumann map of these coefficients provided that are known outside cloaking regions. Moreover, we prove that it can be stably recovered in larger subsets of the domain by enlarging the set of data.

Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 2$, with $\mathscr{C}^\infty$ boundary $\Gamma = \partial\Omega$. Given $T > 0$, we introduce the following dissipative wave equation

$$\begin{cases} \partial_t^2 u - \Delta u + a(x,t)\partial_t u + b(x,t)u = 0 & \text{in} \quad Q = \Omega \times (0,T), \\ u(x,0) = u_0(x), \quad \partial_t u(x,0) = u_1(x) & \text{in} \quad \Omega, \\ u(x,t) = f(x,t) & \text{on} \quad \Sigma = \Gamma \times (0,T), \end{cases} \tag{1}$$

where $f \in H^1(\Sigma)$, $u_0 \in H^1(\Omega)$, $u_1 \in L^2(\Omega)$, and the coefficients $a \in \mathscr{C}^2(Q)$ and $b \in \mathscr{C}^1(Q)$ are assumed to be real valued.

It is clear that with zero initial data, there is no hope to recover a time-dependent coefficient appearing in a hyperbolic equation over the whole cylindrical domain, even from the knowledge of global Neumann data, because the value of the solution can be effected by the value of the initial conditions, which is actually due to a fundamental concept concerning hyperbolic equations called the domain of dependence. Moreover, we can prove that the backward light-cone with base Ω is a cloaking region, that is we can not uniquely recover the coefficients in this region.

In the present work, we address the uniqueness and the stability issues in the study of an inverse problem for the dissipative wave equation (1), in the presence of an absorbing coefficient a and a potential b that depend on both space and time variables. We consider three different sets of data and we aim to show that a and b can be recovered in some specific subsets of the domain, by probing it with disturbances generated on the boundary. The Dirichlet data f is considered as a disturbance that is used to probe the medium which is assumed to be quiet initially.

Keywords: Inverse problems, Dissipative wave equation, Time-dependent coefficients, Stability estimates.
(Joint work with Mourad Bellassoued)

Talk 11. [K. Pankrashkin]
Title: A condition for the absence of resonances at the bottom of essential spectrum for waveguide junctions.
Abstract: Let U be a domain that can be represented as a family of half-infinite cylinders attached to a bounded central domain. We denote by E be the bottom of the essential spectrum of the Dirichlet

Laplacian in U and we say that U admits a resonance at E if there exists a non-trivial bounded solution v of the equation $-\Delta v = Ev$ in U satisfying the Dirichlet condition at the boundary. We prove a condition guaranteeing the absence of resonances in terms of an eigenvalue problem defined on the central domain. This condition will be illustrated by several concrete examples, and we will discuss its applications to the analysis of Laplacians on thin domains converging to branching graphs.

Talk 12. [O. Rouby]
Title: Bohr-Sommerfeld quantization conditions for non-selfadjoint perturbations of selfadjoint operators in dimension one.
Abstract: We interest ourselves in the spectral theory of non-selfadjoint semi-classical operators in dimension one and in asymptotic expansions of eigenvalues. These expansions are written in terms of geometrical objects in a complex phase space coming from classical mechanics and correspond to a generalization of Bohr-Sommerfeld quantization conditions in the non-selfadjoint case. First, we will study non-selfadjoint perturbations of selfadjoint pseudo-differential operators in dimension one. As a corollary, we will establish for PT-symmetric perturbations of selfadjoint operators, that the spectrum is real. Then we will show Bohr-Sommerfeld quantization conditions for non-selfadjoint perturbations of selfadjoint Berezin-Toeplitz operators of the complex plane.

Talk 13. [L. Le Treust]
Title: Asymptotic expansion of eigenvalues for the MIT bag model
Abstract: In this talk we present some spectral asymptotic results of the MIT bag model. This model is the Dirac operator, $-i\alpha \cdot \nabla + m\beta$, defined on a smooth and bounded domain of $\mathbb{R}^3$, Ω, with certain boundary conditions. Specifically, $-i\beta(\alpha \cdot \mathbf{n})\psi = \psi$ must hold at the boundary of Ω, where $\mathbf{n}$ is the outward normal vector and $\psi \in H^1(\Omega, \mathbb{C}^4)$. This model was developed to get a better understanding of the phenomenons involved in the quark-gluon confinement. We study the self-adjointness of the operator and describe the limiting behavior of the eigenvalues of the MIT bag Dirac operator as the mass m tends to $\pm\infty$.

This is a joint work with N. Arrizabalaga and N. Raymond.

Talk 14. [T. Krajewski]
Title: A renormalization group approach to the universality of Wigner's semi-circle law for random matrices with dependent entries.
Abstract: Spectral properties of random matrices have many applications in physics, ranging from nuclear physics to disordered systems. This ubiquity can be traced back to the universality of spectral properties: whatever the distribution of the entries are, the spectral observables obey some universal laws when the size of the matrices become large. A simple example is Wigner's semi-circle law that describes the density of eigenvalues for a hermitian matrix whose entries are independent and identically distributed (iid). We extend it beyond the iid case, provided the cumulants obey a simple power law bound in the size of the matrix. To derive this result, we use the replica technique and a renormalisation group equation for the replica effective action. This is joint work with Vu Dinh Long (student at Ecole Polytechnique) and Adrian Tanasa (LABRI, Bordeaux).
Reference: https://arxiv.org/abs/1609.01873

Talk 15. [M. Mokdad]
Title: Conformal scattering for Maxwell fields on Reissner-Nordstrøm-de Sitter spacetimes.
Abstract: The Reissner-Nordstrøm-de Sitter spacetime models a spherically symmetric charged and non-rotating black hole in the presence of a positive cosmological constant. Depending on the parameters of the metric, this spacetime can have up to three distinct event horizons. In the case of three horizons, we develop a scattering theory for Maxwell fields using the conformal geometric approach initiated by Penrose and Friedlander and referred to as conformal scattering. The idea is that a complete scattering theory is equivalent to the well-posedness of the Goursat problem (characteristic Cauchy problem) at the null boundary of the conformal manifold. Decay estimates

obtained by geometric energy inequalities are essential tools for closing the estimates that allow the construction of the scattering operator : their role is to prove that energy cannot accumulate at timelike infinity, which can be understood as a weak form of Huygens' principle.

Talk 16. [L. Henriet]
Title: Dynamics and topology of a dissipative spin.
Abstract: The notion of topology plays a key role in condensed matter systems, from the study of the hydrodynamic behavior in superfluid helium 3 to the quantization of transport in quantum (spin) Hall systems. In this talk, we analyze the topological deformations of a spin-1/2 in an effective magnetic field induced by an ohmic quantum dissipative environment at zero temperature. This model is known to display a Kosterlitz-Thouless quantum phase transition from a delocalized to a localized phase when increasing the coupling with the environment. From Bethe Ansatz results and a variational approach, we confirm that the Chern number is preserved in the delocalized phase (low coupling) and we report a divergence of the Berry curvature at the equator at the transition. Recent experiments in quantum circuits have engineered non-equilibrium protocols in time to access topological properties at equilibrium from the measure of the (quasadiabatic out-of-equilibrium spin expectation values. Applying a numerically exact stochastic Schrodinger equation we find that, for a fixed sweep velocity, the bath induces a crossover from (quasadiabatic to non-adiabatic dynamical behavior when the spin bath coupling increases. Then, we provide an intuitive physical explanation of the breakdown of adiabaticity in analogy to the Faraday effect in electromagnetism. We demonstrate that the driving of the spin leads to the production of a large number of bosonic excitations in the bath, which in return strongly affect the spin dynamics.

Talk 17. [Z. Ammari]
Title: On the relationship between Hamiltonian dynamics, hierarchies and Liouville's equation
Abstract: I will explain a natural statistical formulation of nonlinear Hamiltonian PDEsin terms of Liouville's equation. Then I will present some recent results that underline the relationship between solutions of Gross-Pitaevskii type hierarchies and Liouville's equation.
(The talk is based on works with F. Nier, Q. Liard and C. Rouffort).